Electrical Engineering 1
12026105

Chapter 5

Operational Amplifier

Learning Objectives

By using the information and exercises in this chapter you will be able to:

1. Comprehend how real operational amplifiers (op amps) function.
2. Understand that ideal op amps function nearly identically to real ones and that they can be used to model them effectively in a variety of circuit applications.
3. Realize how the basic inverting op amp is the workhorse of the op amp family.
4. Use the inverting op amp to create summers.
5. Use the op amp to create a difference amplifier.
6. Explain how to cascade a variety of op amp circuits.

วัตถุประสงค์การเรียนรู้

โดยใช้ข้อมูลและแบบฝึกหัดในบทนี้ นักเรียนจะสามารถ:

1. เข้าใจว่า OP AMP ทำงานอย่างไร
2. เข้าใจการทำงานของ OP AMP ในอุดมคตินั้นจะเหมือนกับ OP AMP ที่ใช้จริงและสามารถใช้ในการสร้างแบบจำลองได้อย่างมีประสิทธิภาพ
3. รู้ว่า OP AMP แบบอินเวอร์ชั่นพื้นฐานเป็นวิธีการทำงานของไฟล์ OP AMP Family
4. ใช้ OP AMP แบบกลับด้านเพื่อวงจรบวก
5. ใช้ OP AMP เพื่อสร้างวงจรแอมพลิฟายเออร์
6. อธิบายวิธีการ OP AMP แบบหลั่น (Cascade)

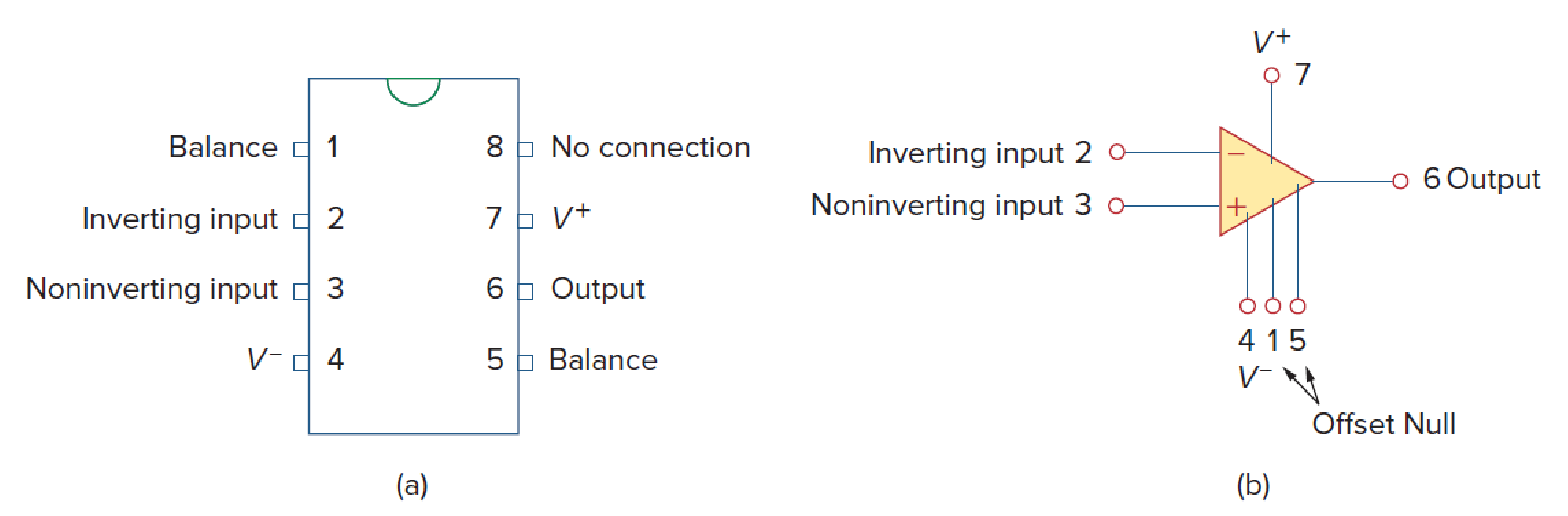
Operational Amplifier - Chapter 5

- What is an Op Amp?
- Ideal Op Amp
- Configuration of Op Amp
- Cascaded Op Amp
- Application : Digital-to Analog Converter

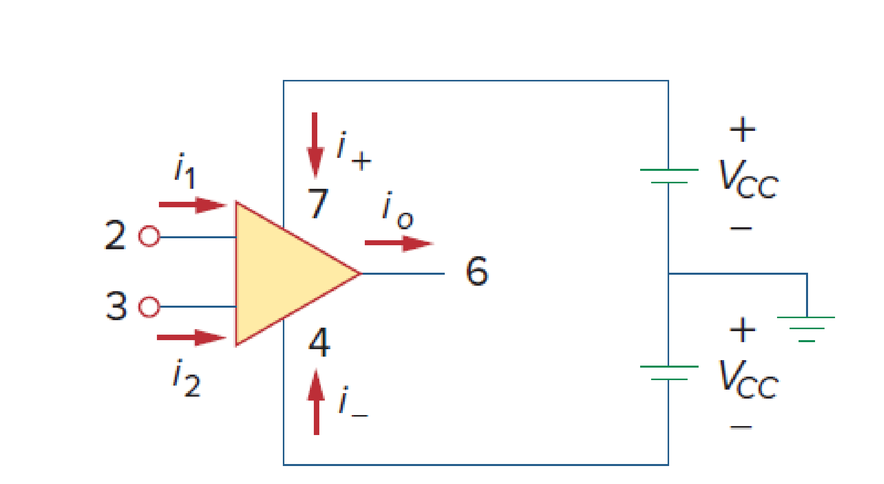
5.1 What is an Op Amp (1)

- It is an electronic unit that behaves like a <u>voltage-controlled voltage source (VCVS)</u>.
- It is an active circuit element designed to perform mathematical operations of *addition*(+), *subtraction*(−), *multiplication*(×), *division*(/), *differentiation*($\frac{d}{dt}$), and *integration*($\int dt$).

5.1 What is an Op Amp (2)



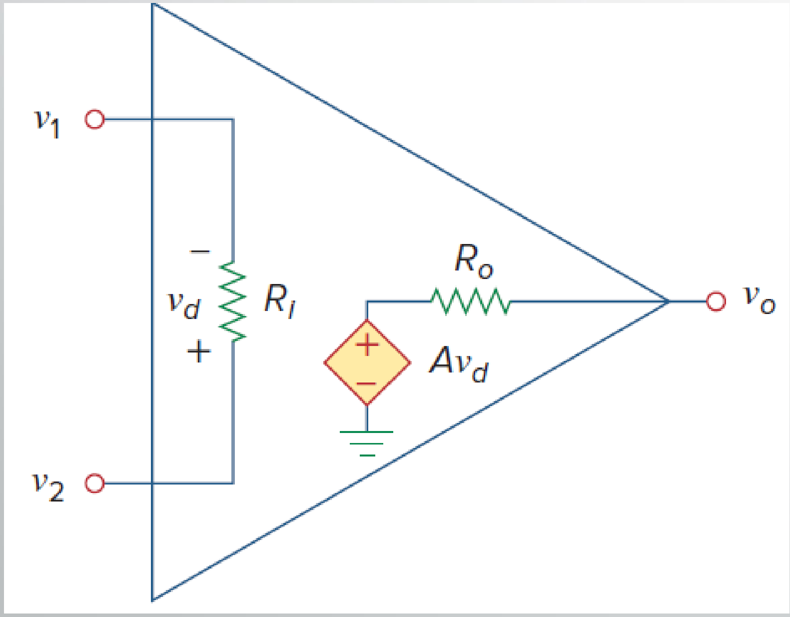
A typical op amp: (a) pin configuration, (b) circuit symbol



5.1 What is an Op Amp (3)

The equivalent circuit

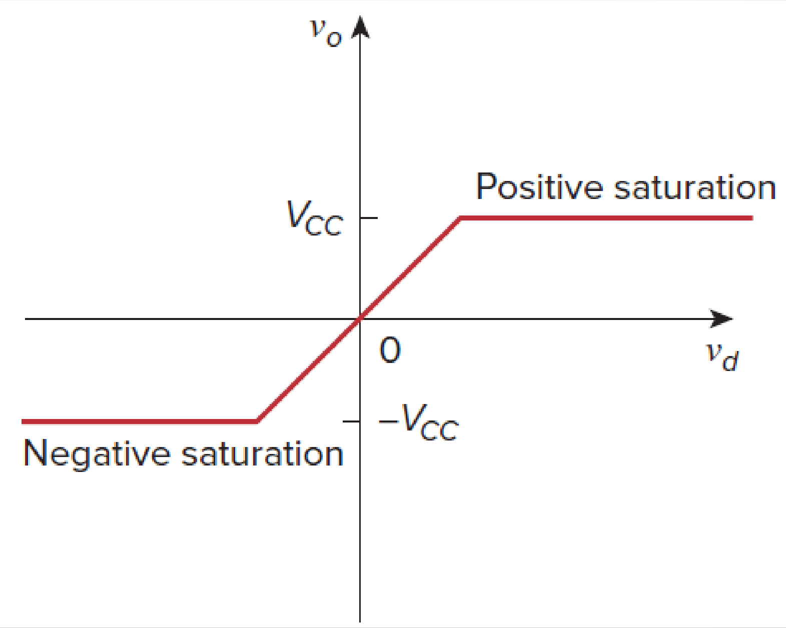
Of the non-ideal op amp



Op Amp output:

v_o as a function of v_d or

$v_o = Av_d$



$$v_d = v_2 - v_1;\quad v_o = Av_d = A(v_2 - v_1)$$

5.1 What is an Op Amp (4)

Typical ranges for op amp parameters

Parameter	Typical range	Ideal values
Open-loop gain, A	10^5 to 10^8 Ω	∞ Ω
Input resistance, R_i	10^5 to 10^{13} Ω	∞ Ω
Output resistance, R_o	10 to 100 Ω	0 Ω
Supply voltage, VCC	5 to 24 V	

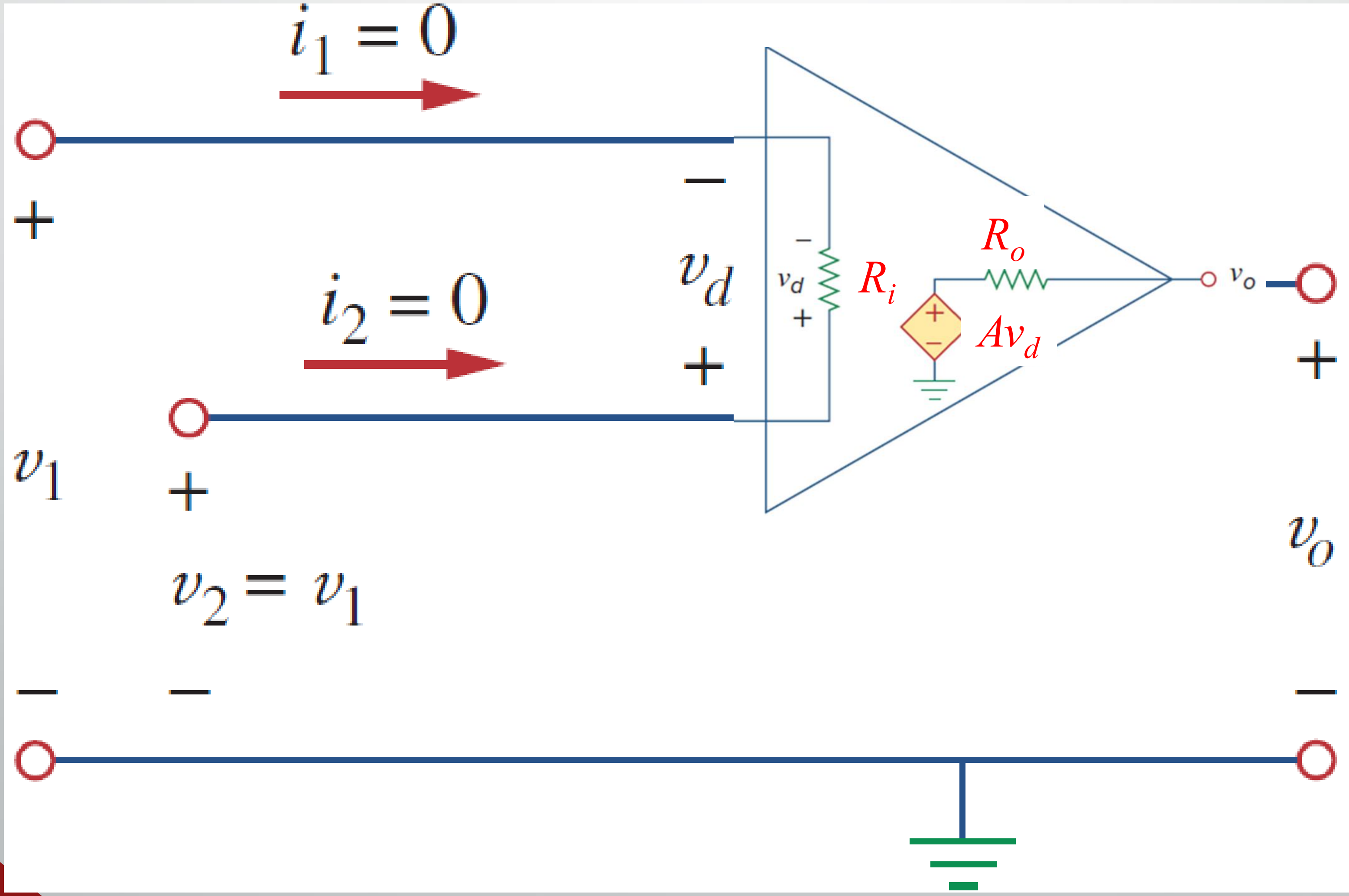
5.2 Ideal Op Amp (1)

An ideal op amp has the following characteristics:

$A \approx \infty$

$R_i \approx \infty$

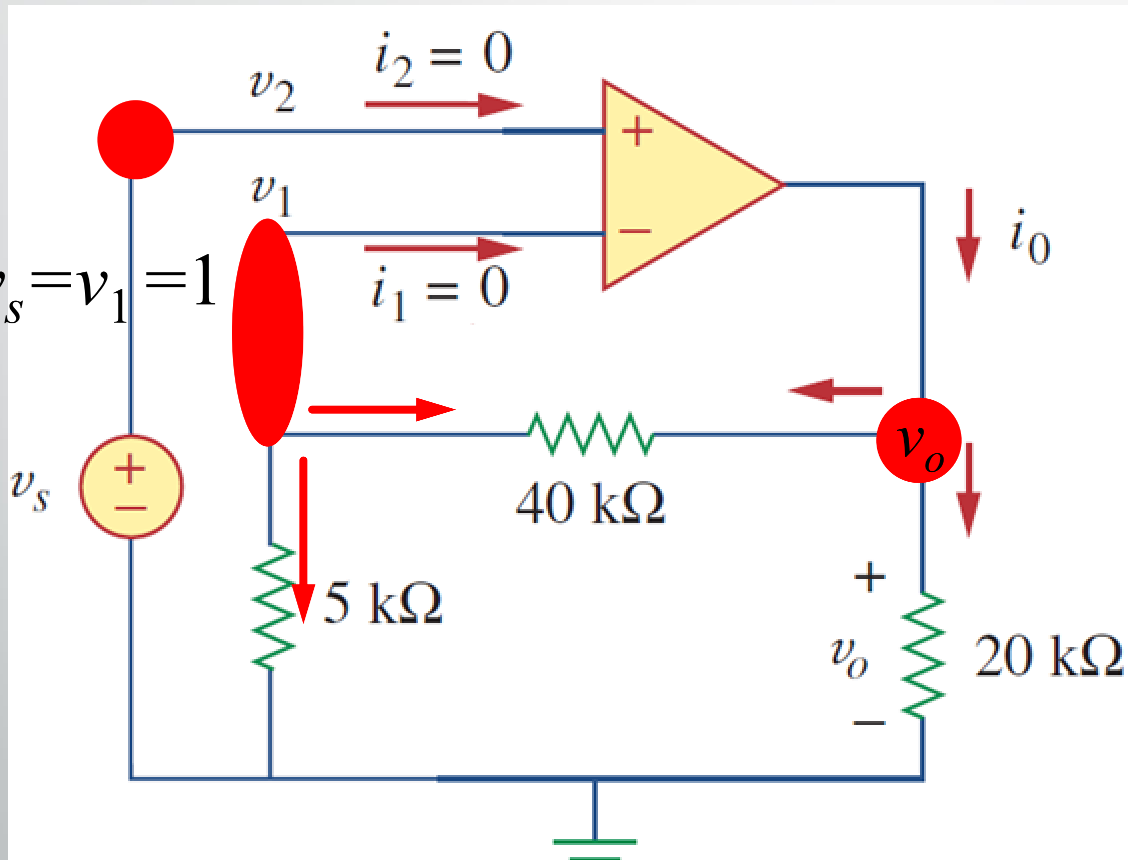
$R_o \approx 0$



5.2 Ideal Op Amp (2)

Ex.1: Determine the value of i_o if v_s=1 V (Hint: Use KCL)

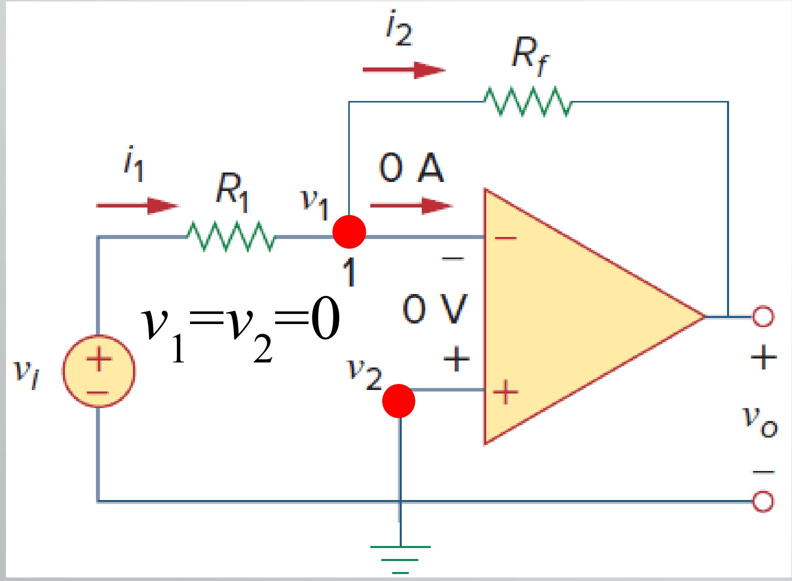
$v_2 = v_s = v_1 = 1$



Ans: 650 μA

5.3 Configuration of Op amp (1)

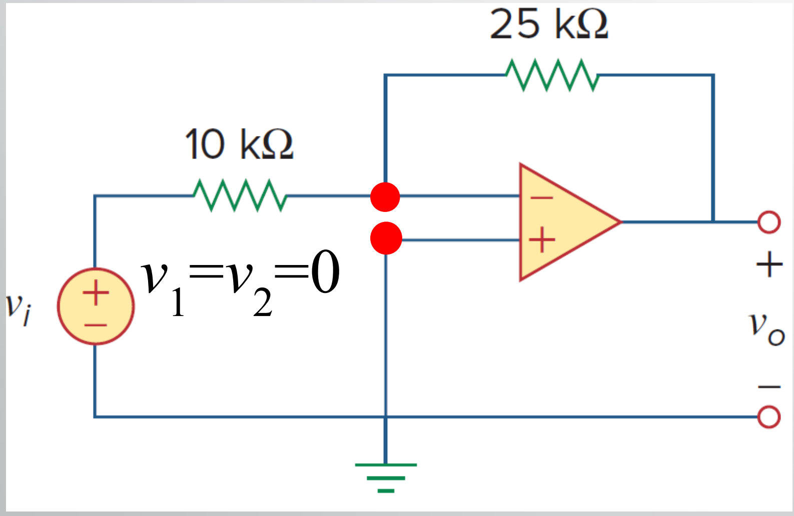
- Inverting amplifier reverses the polarity of the input signal while amplifying it



$$v_o = -\frac{R_f}{R_1} v_i$$

5.3 Configuration of Op amp (2)

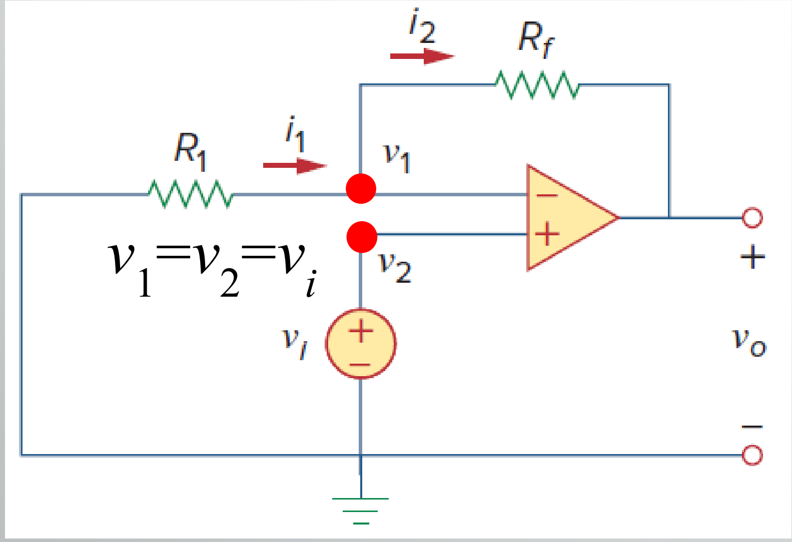
Ex.2 If v_i = 0.5V, calculate: (a) the output voltage, v_o and (b) the current in the 10kΩ resistor.



Ans: (a) -1.25V; (b) 50μA

5.3 Configuration of Op amp (3)

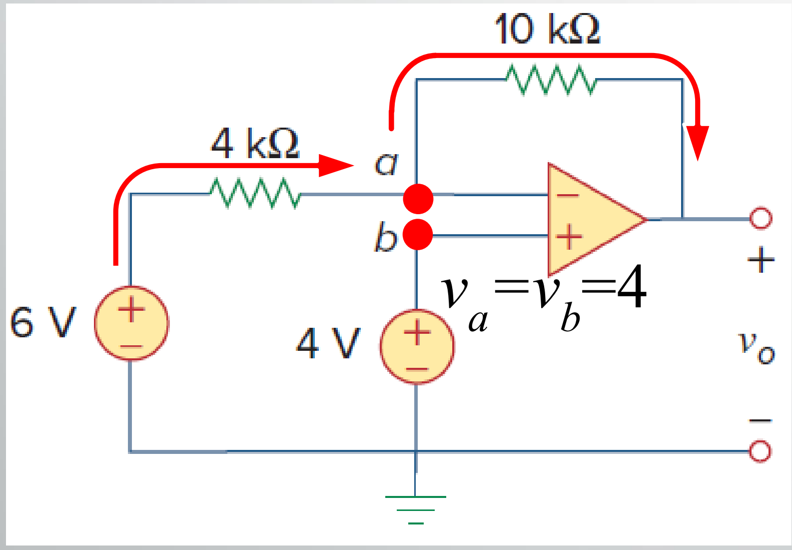
- Non-inverting amplifier is designed to produce positive voltage gain



$$v_o = \left(1 + \frac{R_f}{R_1}\right) v_i$$

5.3 Configuration of Op amp (4)

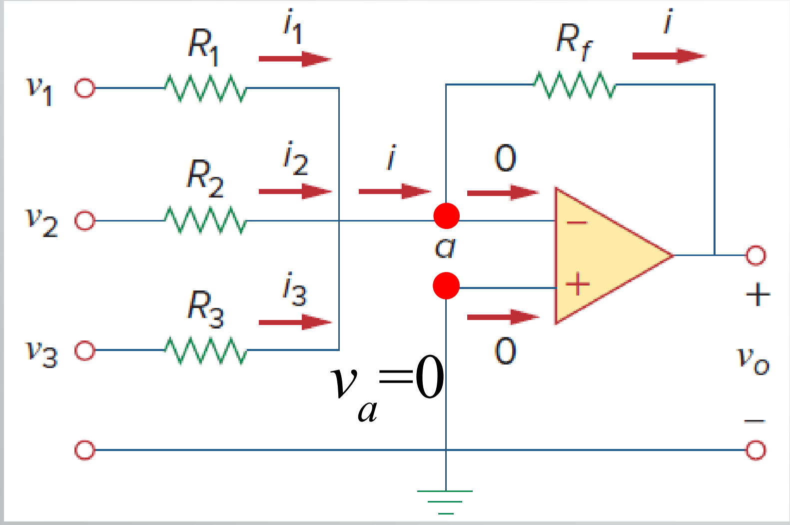
<u>Ex.3</u> For the op amp shown below, calculate the output voltage v_o



Ans: -1V

5.3 Configuration of Op amp (5)

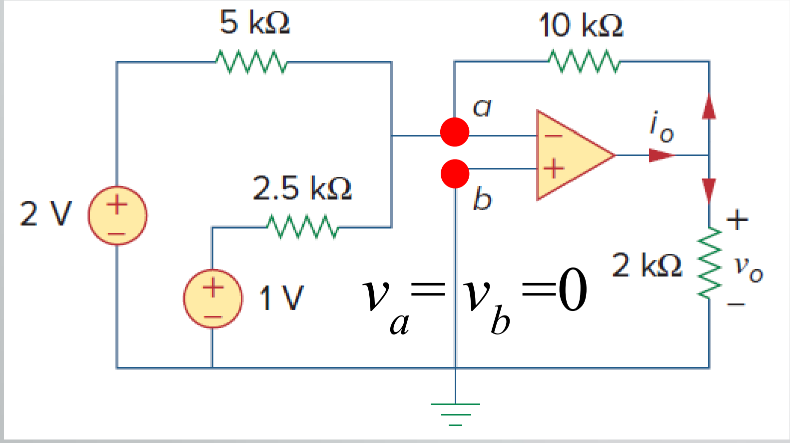
- Summing Amplifier is an op amp circuit that combines several inputs and produces an output that is the weighted sum of the inputs.



$$v_o = -\left(\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2 + \frac{R_f}{R_3} v_3 \right)$$

5.3 Configuration of Op amp (6)

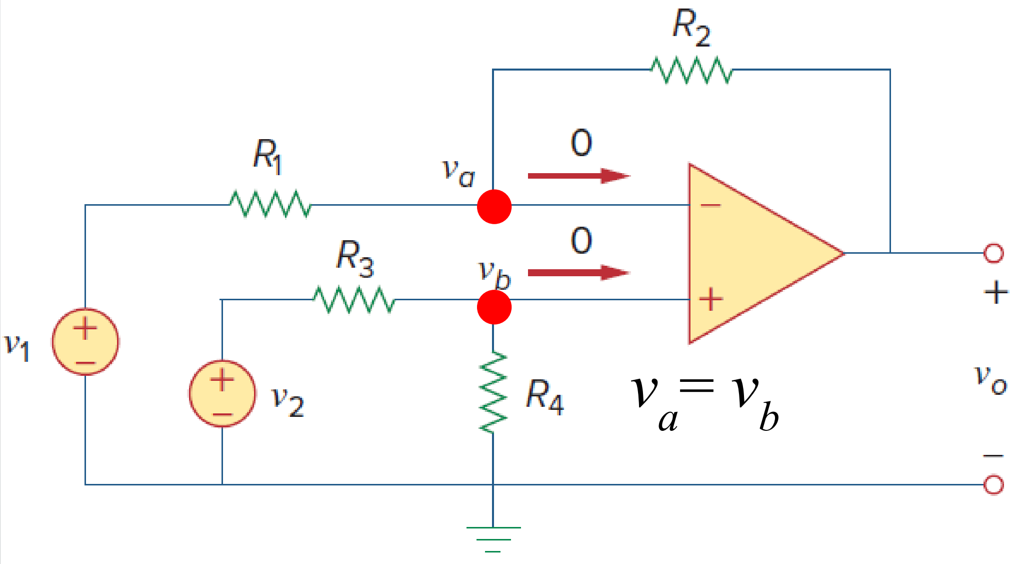
Ex.4 Calculate v_o and i_o in the op amp circuit shown below.



Ans: -8V, -4.8mA

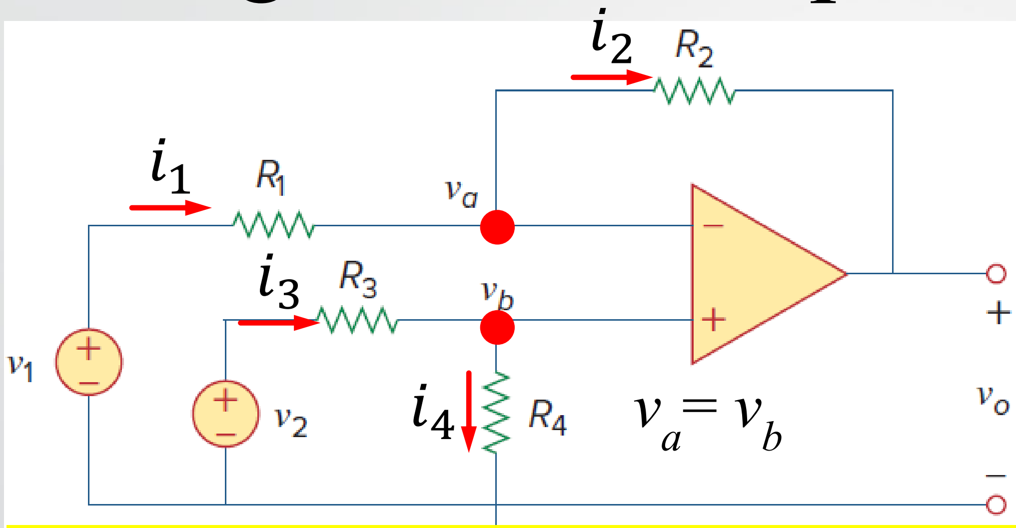
5.3 Configuration of Op amp (7)

Difference amplifier is a device that amplifies the difference between two inputs but rejects any signals common to the two inputs.



$$v_o = \frac{R_2(1 + R_1 / R_2)}{R_1(1 + R_3 / R_4)} v_2 - \frac{R_2}{R_1} v_1 \Rightarrow v_o = v_2 - v_1 \text{ , if } \frac{R_2}{R_1} = \frac{R_3}{R_4} = 1$$

5.3 Configuration of Op amp (7-2)



$$v_o = \frac{R_2(1+R_1/R_2)}{R_1(1+R_3/R_4)}v_2 - \frac{R_2}{R_1}v_1 \Rightarrow v_o = v_2 - v_1 \text{, if } \frac{R_2}{R_1} = \frac{R_3}{R_4} = 1$$

ที่โหนด v_b: $i_3 = i_4$

$$\frac{v_2 - v_a}{R_3} = \frac{v_a}{R_4}$$

$$v_a = \frac{1}{R_3\left(\frac{1}{R_4}+\frac{1}{R_3}\right)}v_2$$

$$v_a = \frac{1}{(1+R_3/R_4)}v_2$$

ที่โหนด v_a: $i_1 = i_2$

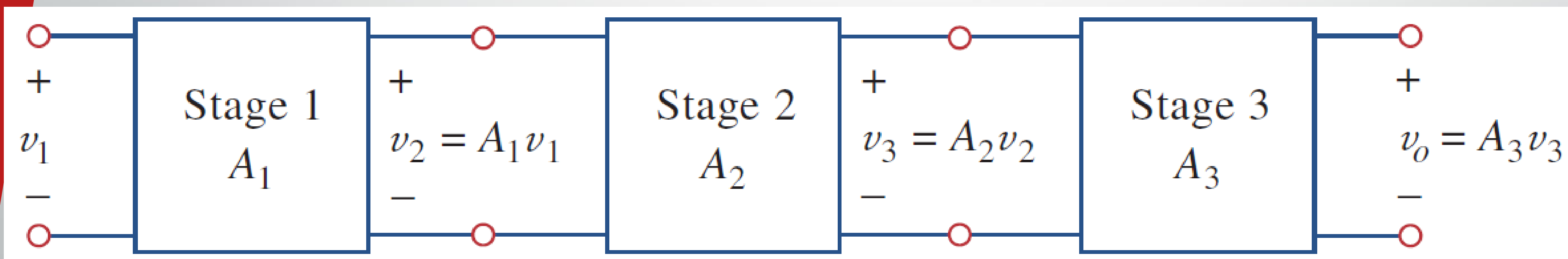
$$\frac{v_1 - v_a}{R_1} = \frac{v_a - v_o}{R_2}$$

$$\frac{v_o}{R_2} = \frac{R_1}{R_1}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)v_a - \frac{v_1}{R_1}$$

$$v_o = \frac{R_2}{R_1}\frac{(1+R_1/R_2)}{(1+R_3/R_4)}v_2 - \frac{R_2}{R_1}v_1$$

5.4 Cascaded Op Amp (1)

- It is a head-to-tail arrangement of two or more op amp circuits such that the output to one is the input of the next.

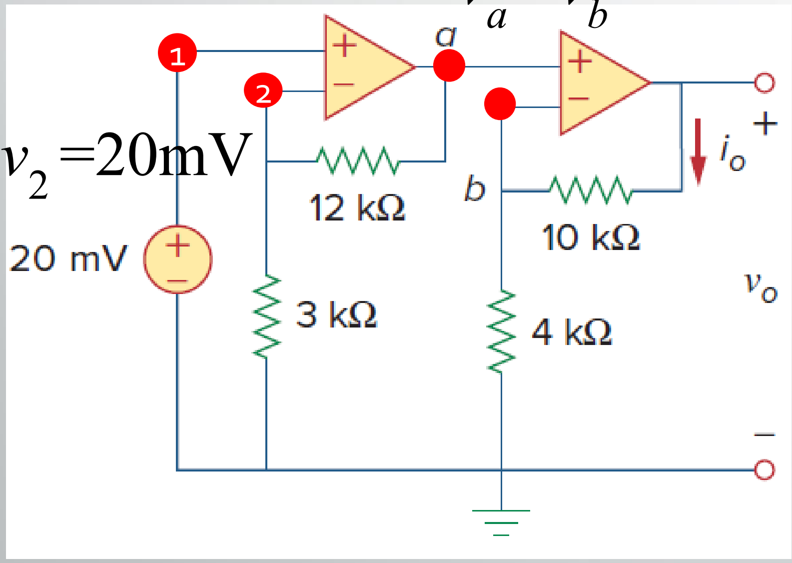


5.4 Cascaded Op Amp (2)

Ex.5 Find v_o and i_o in the circuit shown below.

$v_a = v_b$

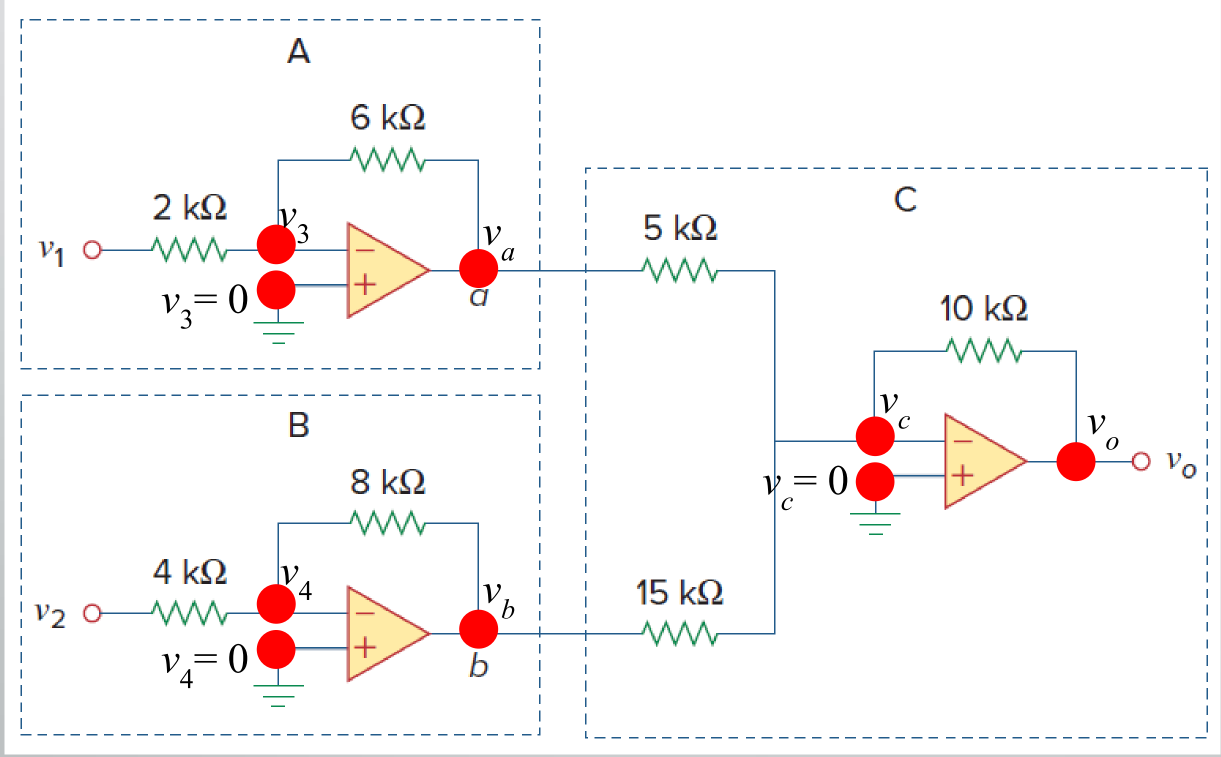
$v_1 = v_2 = 20\text{mV}$



Ans: 350mV, 25μA

5.4 Cascaded Op Amp (3)

Ex.6 If v_1 = 1V and v_2 = 2V, find v_o in the op amp circuit shown below.



Ans: 8.667 V

5.5 Application (1)

- Digital-to Analog Converter (DAC) : it is a device which transforms digital signals into analog form.

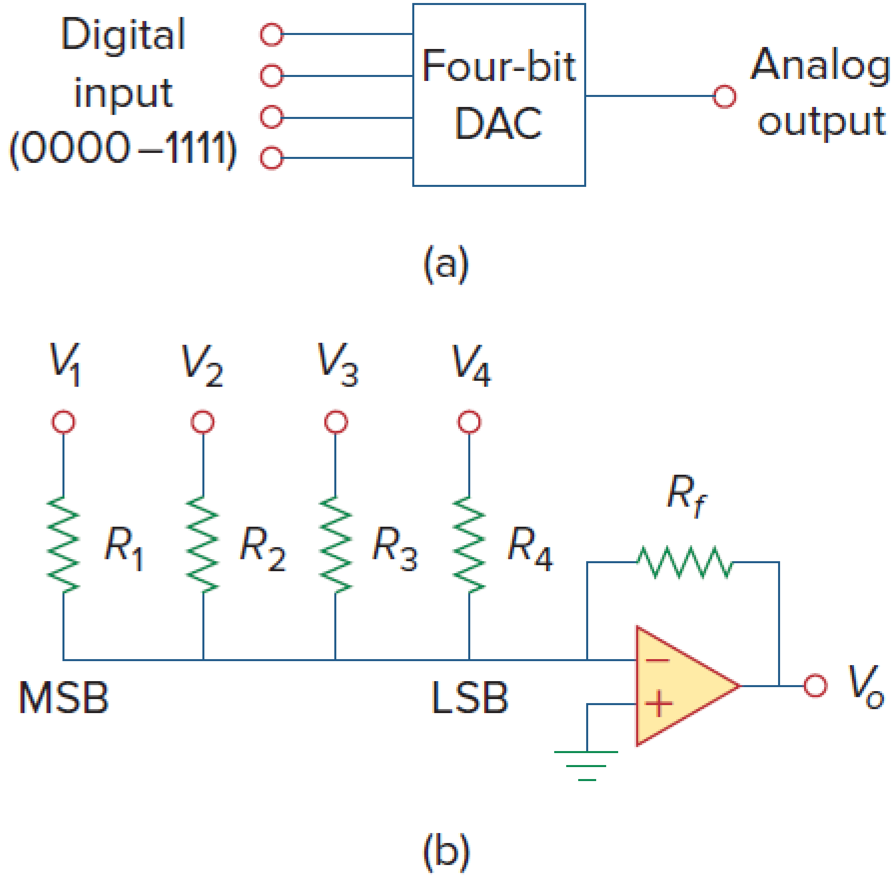


Figure 5.36
Four-bit DAC: (a) block diagram, (b) binary weighted ladder type.

$$-V_0 = \frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3 + \frac{R_f}{R_4}V_4$$

where

V1 most significant bit (MSB),

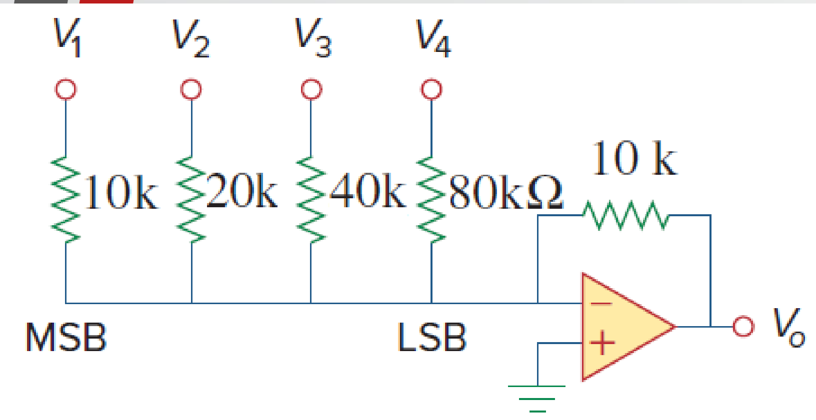
V4 least significant bit (LSB).

V1 to V4 are either 0 or 1 V

- By using the proper input and feedback resistor values, the DAC provides a single output that is proportional to the inputs.

5.5 Application(2)

Ex.7 Circuit shown below, Obtain the analog output for binary inputs [0000], [0001], [0010],…, [1111].



$$-V_0 = \frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3 + \frac{R_f}{R_4}V_4$$

$$-V_o = V_1 + 0.5V_2 + 0.25V_3 + 0.125V_4$$

a digital input $[V_1V_2V_3V_4]$ = [0000] produces an analog output of $-V_o$ = 0 V
a digital input $[V_1V_2V_3V_4]$ = [0001] produces an analog output of $-V_o$ = 0.125 V
a digital input $[V_1V_2V_3V_4]$ = [0010] produces an analog output of $-V_o$ = 0.25 V
a digital input $[V_1V_2V_3V_4]$ = [0011] produces an analog output of $-V_o$ = 0.375 V
a digital input $[V_1V_2V_3V_4]$ = [0100] produces an analog output of $-V_o$ = 0.5 V
a digital input $[V_1V_2V_3V_4]$ = [0101] produces an analog output of $-V_o$ = 0.625 V
a digital input $[V_1V_2V_3V_4]$ = [0110] produces an analog output of $-V_o$ = 0.75 V
a digital input $[V_1V_2V_3V_4]$ = [0111] produces an analog output of $-V_o$ = 0.875 V
a digital input $[V_1V_2V_3V_4]$ = [1000] produces an analog output of $-V_o$ = 1 V
a digital input $[V_1V_2V_3V_4]$ = [1001] produces an analog output of $-V_o$ = 1.125 V
a digital input $[V_1V_2V_3V_4]$ = [1010] produces an analog output of $-V_o$ = 1.25 V
a digital input $[V_1V_2V_3V_4]$ = [1011] produces an analog output of $-V_o$ = 1.375 V
a digital input $[V_1V_2V_3V_4]$ = [1100] produces an analog output of $-V_o$ = 1.5 V
a digital input $[V_1V_2V_3V_4]$ = [1101] produces an analog output of $-V_o$ = 1.625 V
a digital input $[V_1V_2V_3V_4]$ = [1110] produces an analog output of $-V_o$ = 1.75 V
a digital input $[V_1V_2V_3V_4]$ = [1111] produces an analog output of $-V_o$ = 1.875 V

5.5 Application(3)

TABLE 5.2

Input and output values of the four-bit DAC.

Binary input $[V_1V_2V_3V_4]$	Decimal value	Output $-V_o$
0000	0	0
0001	1	0.125
0010	2	0.25
0011	3	0.375
0100	4	0.5
0101	5	0.625
0110	6	0.75
0111	7	0.875
1000	8	1.0
1001	9	1.125
1010	10	1.25
1011	11	1.375
1100	12	1.5
1101	13	1.625
1110	14	1.75
1111	15	1.875

5.5 Application(4)

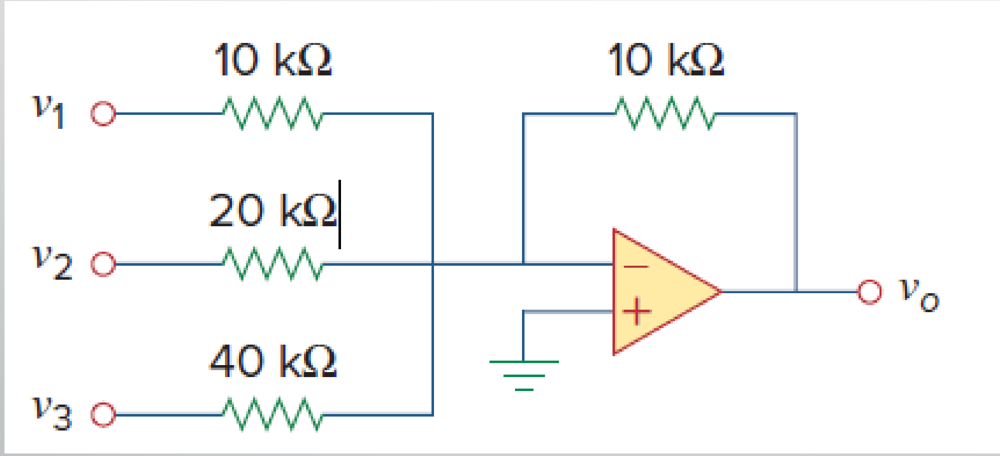
<u>Ex.8</u> 3-bit DAC is shown in Figure.

(a) Determine $|v_o|$ for $[v_1\ v_2\ v_3]$ = [010].

(b) Find |Vo| if $[v_1\ v_2\ v_3]$ = [110].

(c) If $|v_o|$ = 1.25 V is desired, what should be $[v_1\ v_2\ v_3]$?

(d) To get $|v_o|$ = 1.75 V, what should be $[v_1\ v_2\ v_3]$?



Ans: 0.5 V, 1.5 V, [101], [111].

Capacitors and Inductors Chapter 6

6.1 Introduction

6.2 Capacitors (ตัวเก็บประจุ)

6.3 Series and Parallel Capacitors (ต่ออนุกรม,ขนานตัวเก็บประจุ)

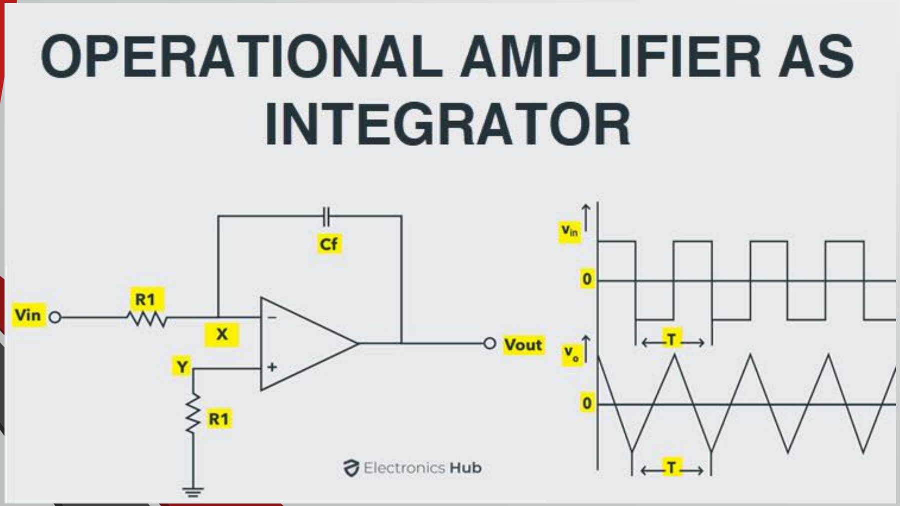
6.4 Inductors (ขดลวดเหนี่ยวนำ)

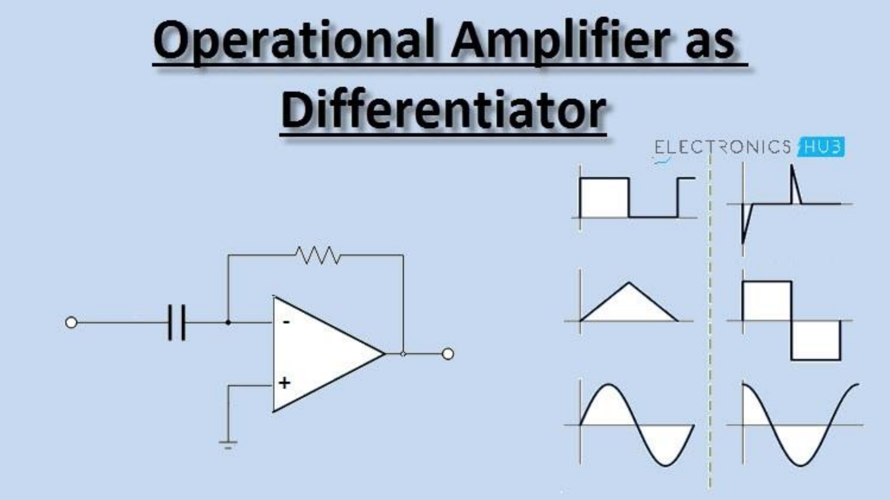
6.5 Series and Parallel Inductors (ต่ออนุกรม,ขนานขดลวดเหนี่ยวนำ)

6.6 Applications

6.1 Introduction

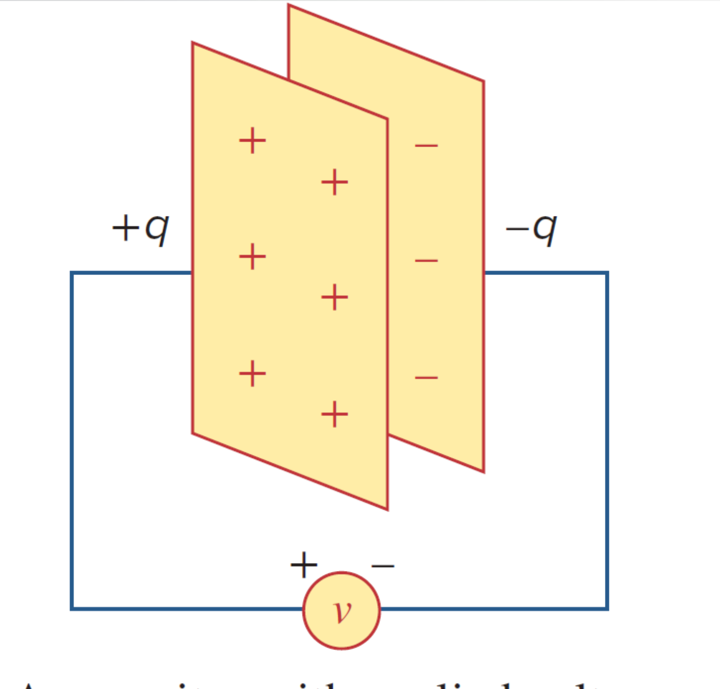
- Unlike resistors, which dissipate energy, capacitors and inductors do not dissipate but store energy, which can be retrieved later.
- For this reason, capacitors and inductors are called storage elements.
- As typical applications, we explore how capacitors are combined with op amps to form integrators, differentiators, and analog computers.



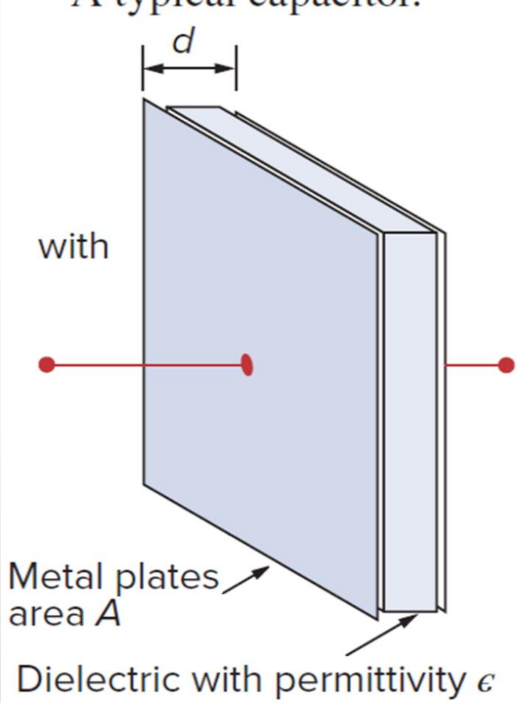


6.2 Capacitors (1)

- A capacitor is a passive element designed to **store energy** in its **electric field.**



A capacitor with applied voltage v.

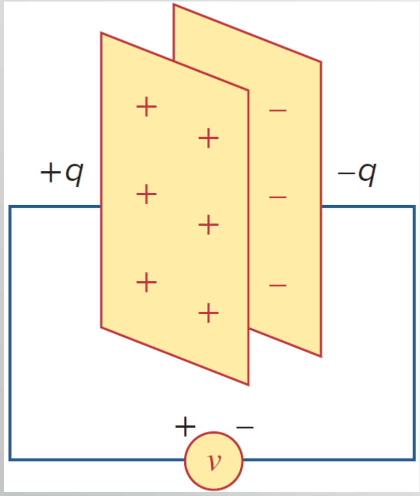


A typical capacitor.

- A **capacitor** consists of two conducting plates separated by an insulator (or dielectric).

6.2 Capacitors (3)

- **Capacitance** *C* is the ratio of the charge *q* on one plate of a capacitor to the voltage difference *V* between the two plates, measured in farads (*F*).

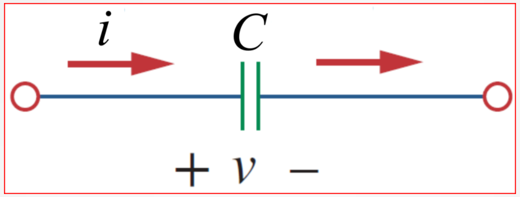


$$i = C\frac{dv}{dt} \quad \text{and} \quad C = \frac{q}{V} = \frac{\varepsilon A}{d}$$

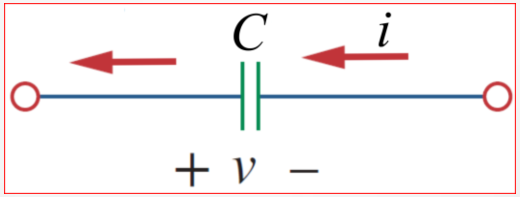
- Where ε is the permittivity of the dielectric material between the plates, A is the surface area of each plate, d is the distance between the plates. ($\varepsilon_0 = 8.85 \times 10^{-12}$ F/m)
- Unit: F, pF (10^{-12}), nF (10^{-9}), and μF (10^{-6})

6.2 Capacitors (4)

- If *i* is flowing into the positive (+) terminal of *C*
  - Charging => *i* is positive (+)



  - Discharging => *i* is negative (-)



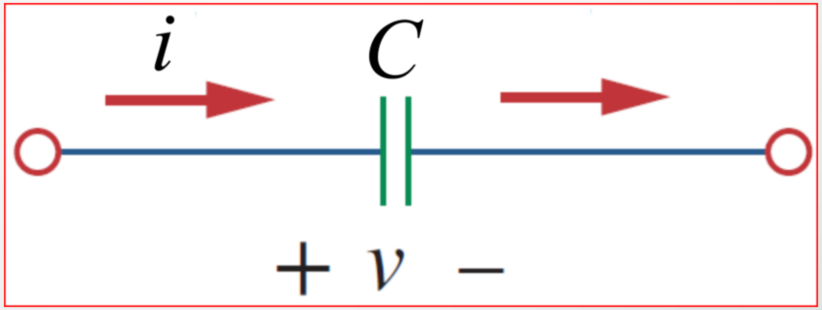
- The *i-v* relationship of capacitor according to above convention is

$$i = C\frac{dv}{dt}$$ and $$v = \frac{1}{C}\int_{t_0}^{t} i\,d\,t + v(t_0)$$

6.2 Capacitors (5)

- The energy, ***w***, stored in the capacitor is

$$w = \frac{1}{2} C v^2$$



- A capacitor is
  - an **open circuit** to dc (กระแสตรง)
  - its voltage **cannot change abruptly**.

$$(i = C\frac{dv}{dt} = 0)$$

6.2 Capacitors (6)

Ex.1 The current through a 100-μF capacitor is $i(t)$ = $50\sin(120\pi t)$ mA. Calculate the voltage across it at t =1 ms and t = 5 ms. Take $v(t_0)$= $v(0)$= 0.

$$v = \frac{1}{C}\int_{t_0}^{t} i\,dt + v(t_0)$$

$$v = \frac{50mA}{100\mu F}\int_0^{1ms} \sin(120\square t)\,dt = 500\left(-\frac{\cos(120\pi t)}{120\pi}\bigg|_0^{1ms}\right) = 1.3263(1-\cos(0.377))$$
$$= 93.14mV$$

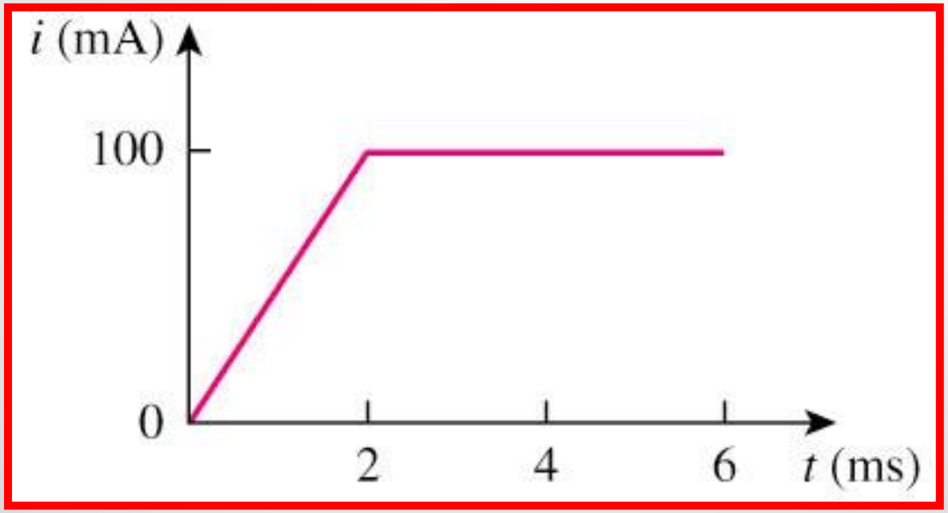
$$v = \frac{50mA}{100\mu F}\int_0^{5ms} \sin(120\square t)\,dt = 500\left(-\frac{\cos(120\pi t)}{120\pi}\bigg|_0^{5ms}\right) = 1.3263(1-\cos(1.885))$$
$$= 1.7361V$$

Answer: v(1ms) = 93.14mV , v(5ms) = 1.7361V

6.2 Capacitors (7)

<u>Ex.2</u> An initially uncharged 1mF capacitor has the current shown below across it. Calculate the voltage across it at

$t = 2$ ms and $t = 5$ ms.



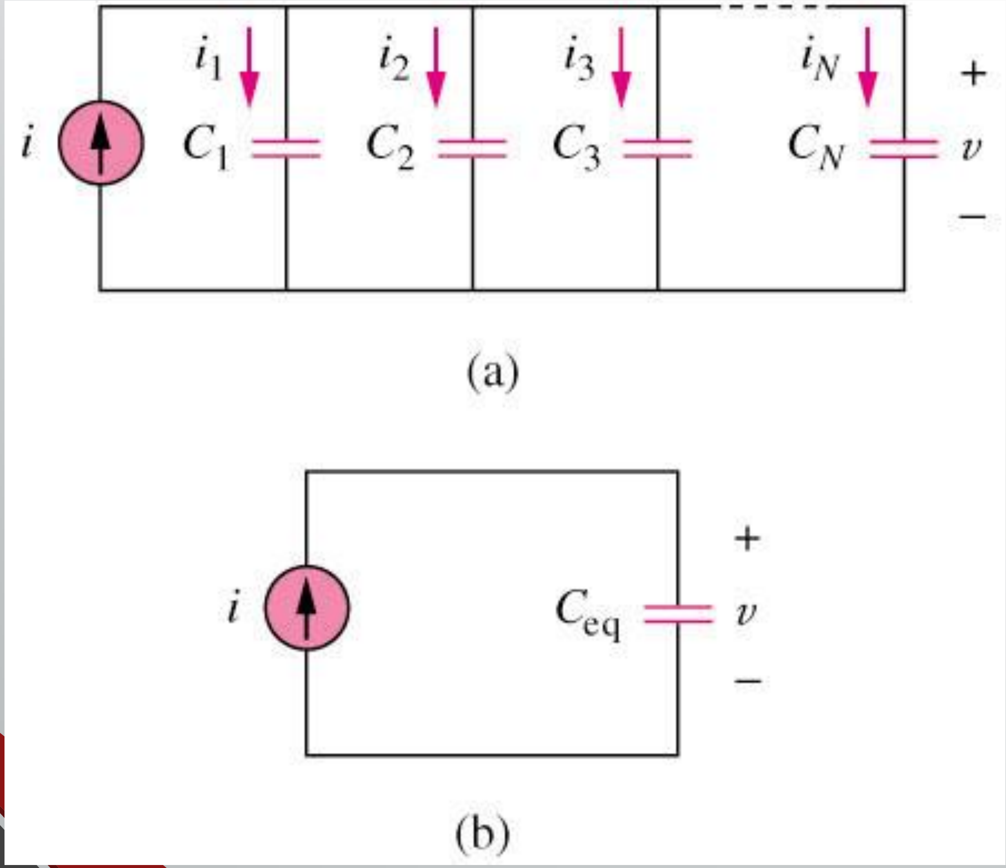
$$v = \frac{1}{1\text{mF}}\int_0^{2ms} 50t\,dt = 1000\left(25t^2\Big|_0^{2ms}\right) = 0.1V = 100mV$$

$$v = \frac{1}{1\text{mF}}\int_{2ms}^{5ms} 100m\,dt + 100mV = 100(3mV) + 100mV = 400mV$$

Answer: v(2ms) = 100 mV , v(5ms) = 400 mV

6.3 Series and Parallel Capacitors (1)

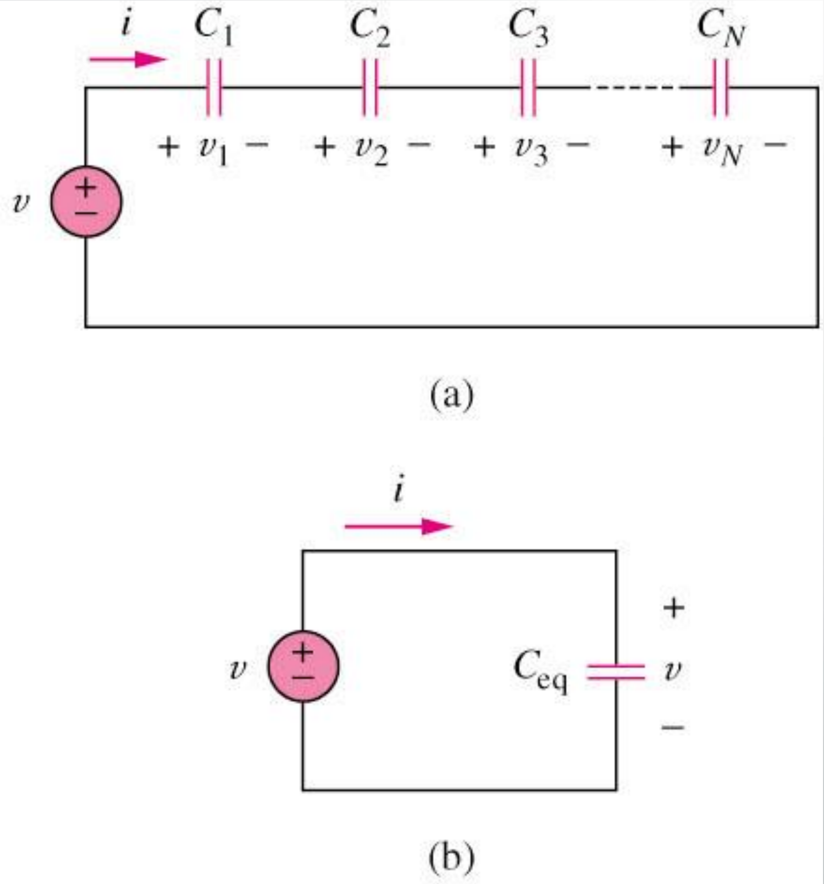
- The equivalent capacitance of N **parallel-connected** capacitors is the sum of the individual capacitances.



$$C_{eq} = C_1 + C_2 + ... + C_N$$

6.3 Series and Parallel Capacitors (2)

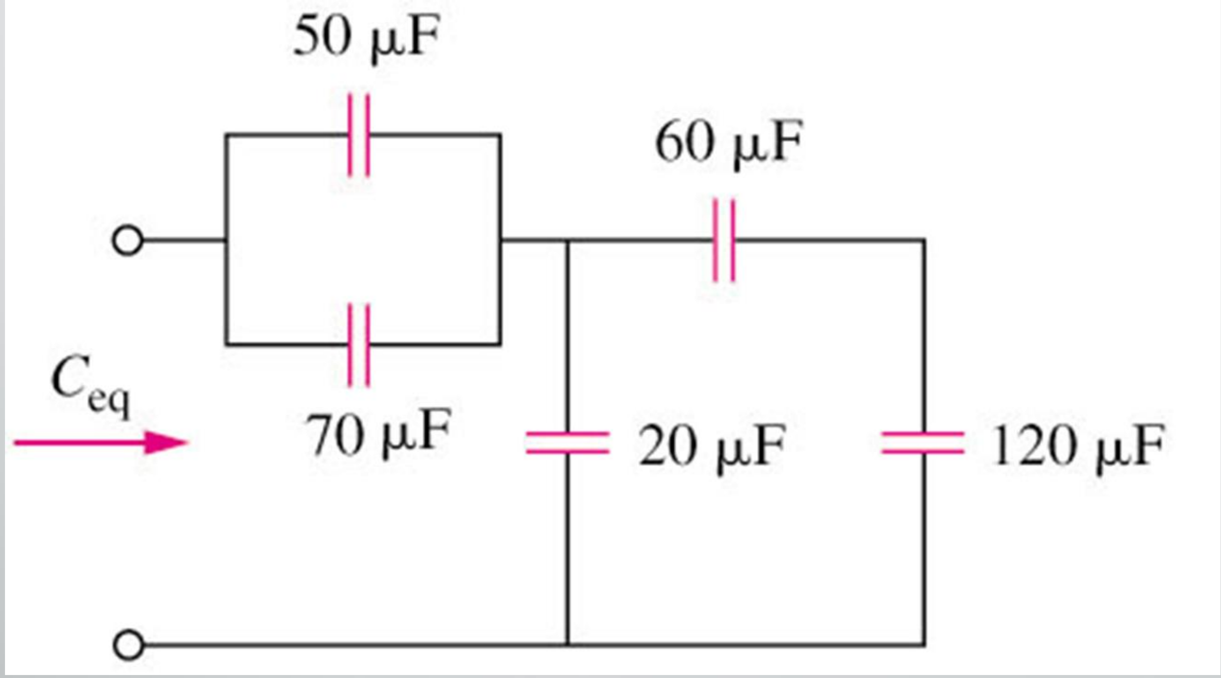
- The equivalent capacitance of *N* **series-connected** capacitors is the reciprocal of the sum of the reciprocals of the individual capacitances.



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_N}$$

6.3 Series and Parallel Capacitors (3)

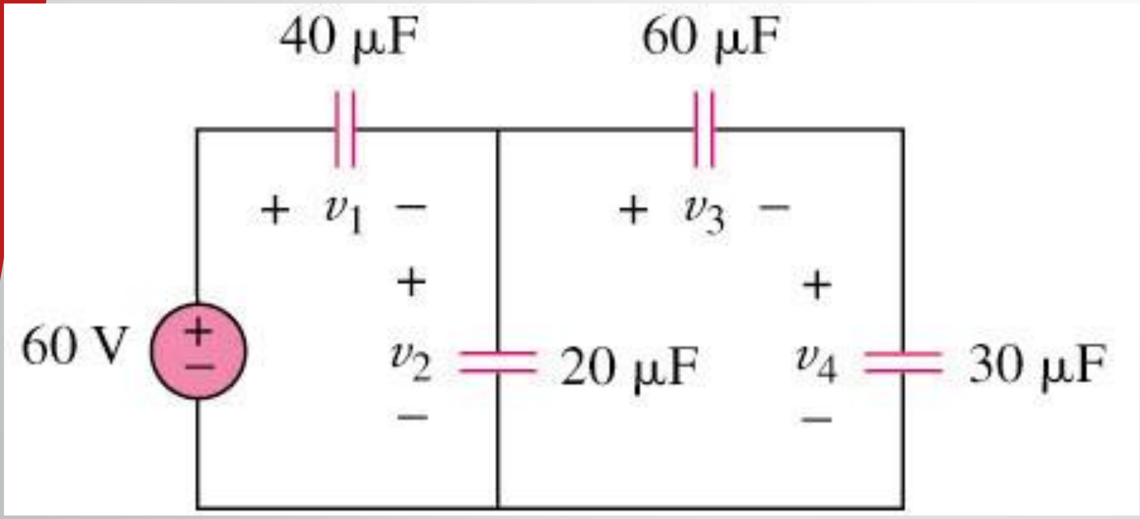
Ex.3 Find the equivalent capacitance seen at the terminals of the circuit in the circuit shown below:

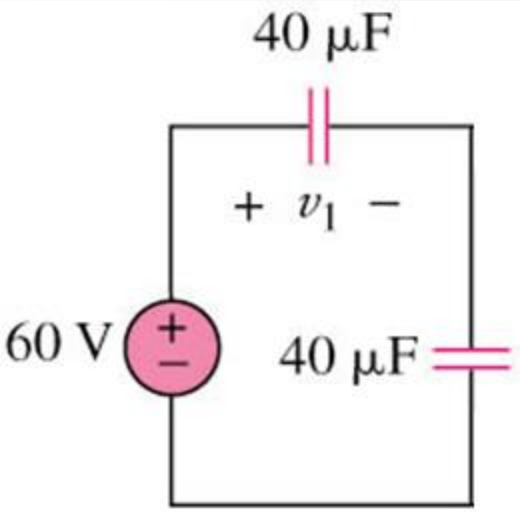


Answer: C_{eq} = 40μF

6.3 Series and Parallel Capacitors (4)

Ex.4 Find the voltage across each of the capacitors in the circuit shown below:

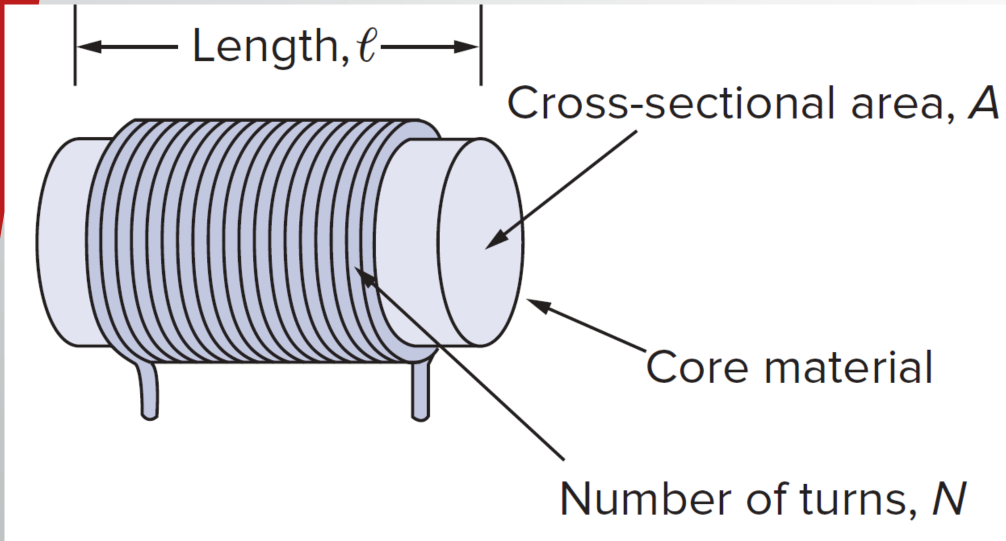




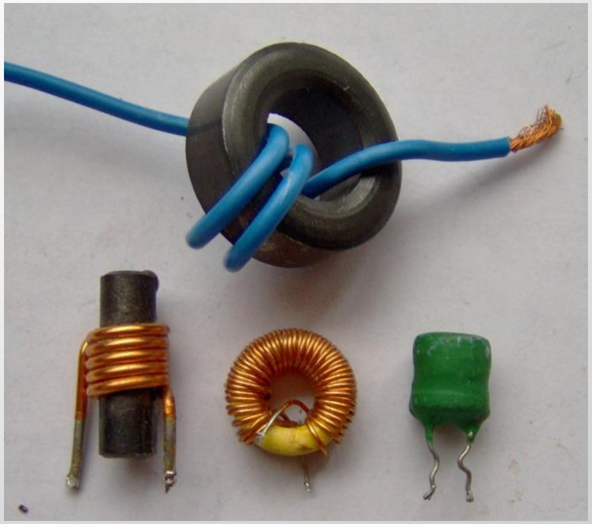
Answer: v_1 = 30V, v_2 = 30V, v_3 = 10V, v_4 = 20V

6.4 Inductors (1)

- An inductor is a passive element designed to store energy in its magnetic field.



Typical form of an inductor.



- An inductor consists of a coil of conducting wire.

6.4 Inductors (2)

- Inductance is the property whereby an inductor exhibits
- opposition to the change of current flowing through it, measured in henrys (H). ($\mu_0 = 4\boldsymbol{\pi}\times 10^{-7}$ H/m)

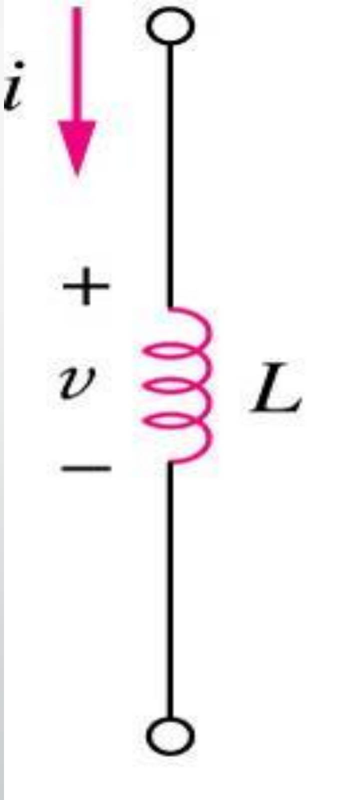
$$v = L\frac{di}{dt}$$ and $$L = \frac{\mu A N^2}{l}$$

- The unit of inductors is Henry (H), mH (10^{-3}) and nH (10^{-9}).

6.4 Inductors (3)

- The current-voltage relationship of an inductor:

$$i = \frac{1}{L}\int_{t_0}^{t} v(t)\,dt + i(t_0)$$



- The power stored by an inductor:

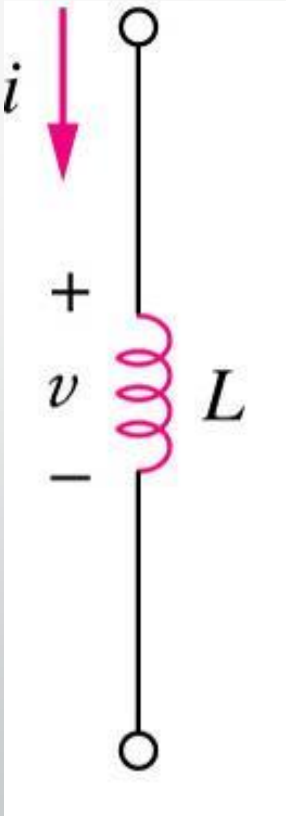
$$w = \frac{1}{2}Li^2$$

- An inductor acts like a short circuit to dc (di/dt = 0) and its current cannot change abruptly.

6.4 Inductors (4)

Ex.5 The terminal voltage of a 2 H inductor is v = 10(1-t) V

Find the current flowing through it at t = 4 s and the energy stored in it within 0 < t < 4 s. Assume i(0) = 2 A.



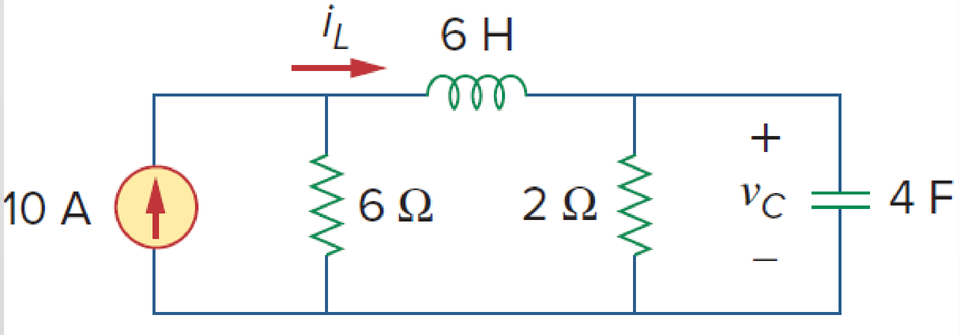
$$i = \frac{1}{2}\int_0^4 10(1-t)\,dt + 2 = \frac{1}{2}\left(10t - 5t^2\Big|_0^4\right) + 2 = -20 + 2 = -18A$$

$$w = \frac{1}{2}Li^2 - \frac{1}{2}Li_o^2 = \frac{1}{2}(2)(-18)^2 - 4 = 324\text{J} - 4\text{J} = 320\text{J}$$

Answer: i(4s) = -18A , w(4s) = 320J

6.4 Inductors (5)

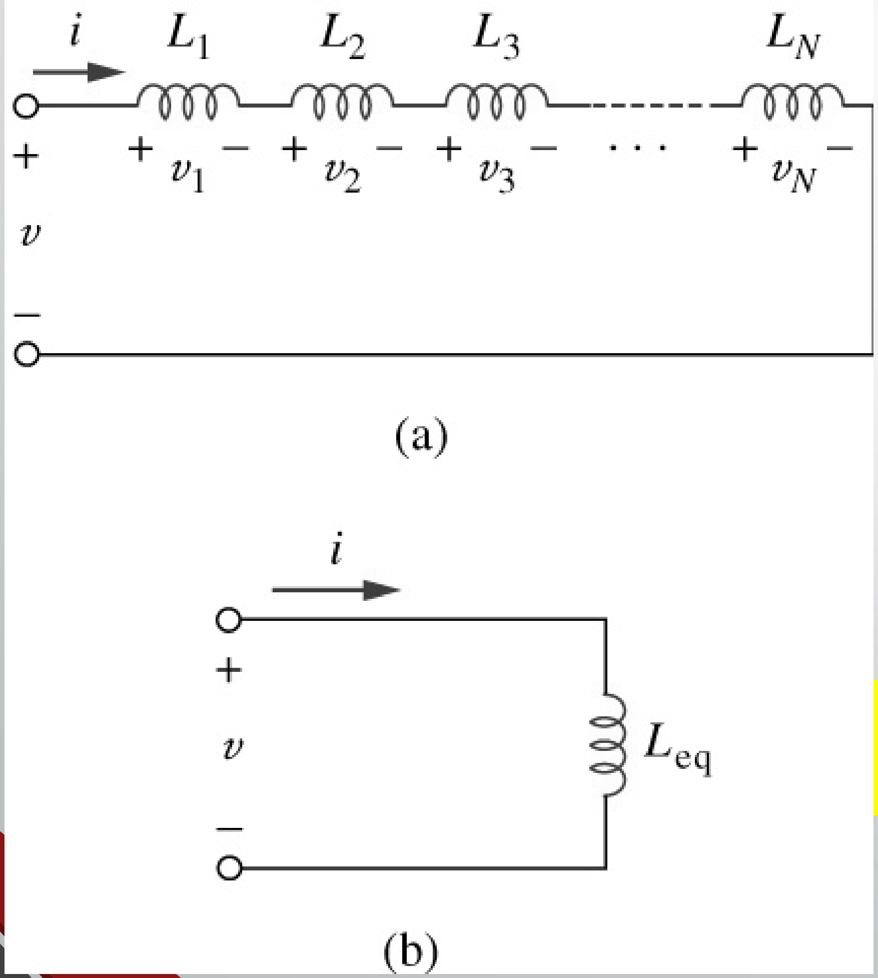
Ex.6 Determine v_c, i_L, and the energy stored in the capacitor and inductor in the circuit shown below under dc conditions.



$i_L = 7.5A,\ v_C = 15V,\ w_C = 450\text{J},\ w_L = 168.75\text{J}$

6.5 Series and Parallel Inductors (1)

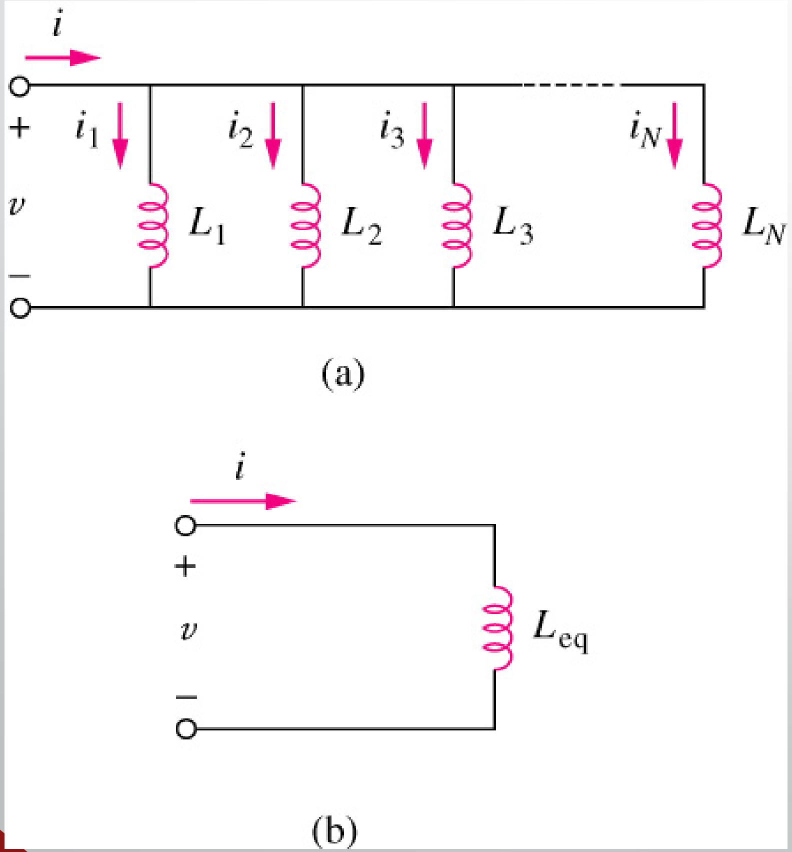
- The equivalent inductance of **series-connected** inductors is the sum of the individual inductances.



$$L_{eq} = L_1 + L_2 + \dots + L_N$$

6.5 Series and Parallel Inductors (2)

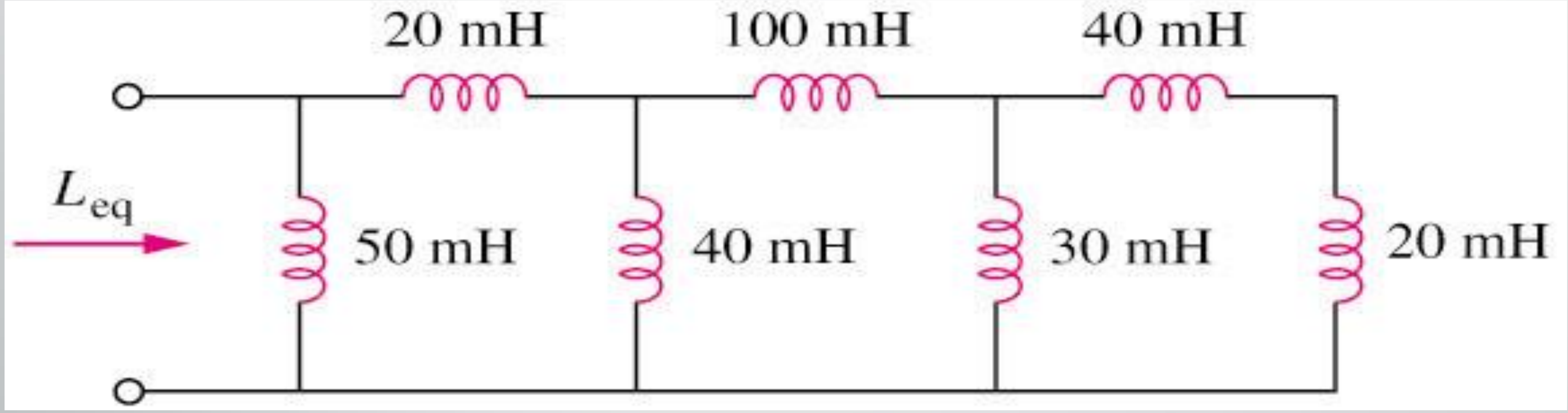
- The equivalent capacitance of **parallel** inductors is the reciprocal of the sum of the reciprocals of the individual inductances.



$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots + \frac{1}{L_N}$$

6.5 Series and Parallel Capacitors (3)

Ex.7 Calculate the equivalent inductance for the inductive ladder network in the circuit shown below:

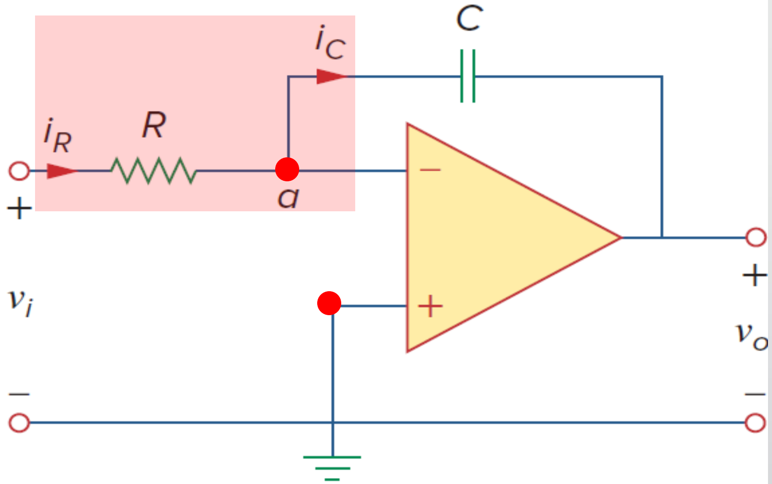


Answer: L_{eq} = 25mH

6.6 Applications (Integrator)

An **integrator** is an op amp circuit whose output is proportional to the integral of the input signal.

It is interesting that we can obtain a mathematical representation of integration this way. At node *a* in Fig.



$$i_R = i_C$$

But

$$i_R = \frac{v_i}{R}, \qquad i_C = -C\frac{dv_o}{dt}$$

Substituting these in Eq., we obtain

$$\frac{v_i}{R} = -C\frac{dv_o}{dt}$$

$$dv_o = -\frac{1}{RC}v_i\,dt$$

Integrating both sides gives

$$v_o(t) - v_o(0) = -\frac{1}{RC}\int_0^t v_i(\tau)\,d\tau$$

To ensure that $v_o(0) = 0$, it is always necessary to discharge the integrator's capacitor prior to the application of a signal. Assuming $v_o(0) = 0$,

$$v_o = -\frac{1}{RC}\int_0^t v_i(\tau)\,d\tau$$

6.6 Applications (Differentiator)

A **differentiator** is an op amp circuit whose output is proportional to the rate of change of the input signal.

Applying KCL at node a,

$$i_R = i_C$$

But $\quad i_R = -\frac{v_o}{R}, \qquad i_C = C\frac{dv_i}{dt}$

Substituting these in Eq. yields

$$v_o = -RC\frac{dv_i}{dt}$$

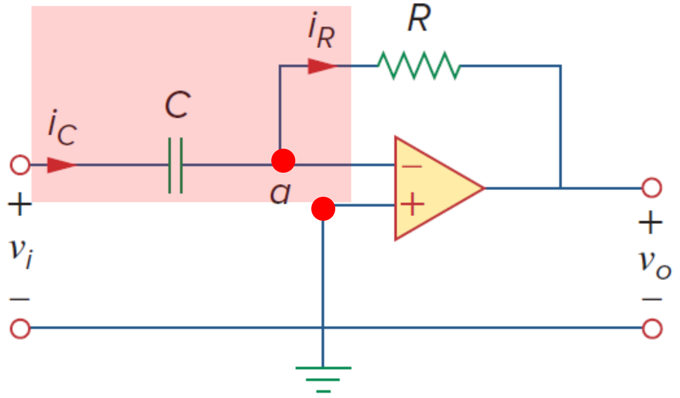


TABLE 6.1

Important characteristics of the basic elements.†

Relation	Resistor (R)	Capacitor (C)	Inductor (L)
v-i:	$v = iR$	$v = \frac{1}{C}\int_{t_0}^{t} i(\tau)\, d\tau + v(t_0)$	$v = L\frac{di}{dt}$
i-v:	$i = v/R$	$i = C\frac{dv}{dt}$	$i = \frac{1}{L}\int_{t_0}^{t} v(\tau)\, d\tau + i(t_0)$
p or w:	$p = i^2R = \frac{v^2}{R}$	$w = \frac{1}{2}Cv^2$	$w = \frac{1}{2}Li^2$
Series:	$R_{eq} = R_1 + R_2$	$C_{eq} = \frac{C_1C_2}{C_1 + C_2}$	$L_{eq} = L_1 + L_2$
Parallel:	$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$	$C_{eq} = C_1 + C_2$	$L_{eq} = \frac{L_1L_2}{L_1 + L_2}$
At dc:	Same	Open circuit	Short circuit
Circuit variable that cannot change abruptly:	Not applicable	v	i